Damage rheology applied to ice stream margin dynamics

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Damage mechanics – p.2/16

Damage mechanics

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- can lead to oscillatory behaviour

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Damage evolution law (viscous):

$$\frac{d\omega}{dt} = F_{\text{visc}}(J_1, J_2, \omega)$$

Elastic damage law

Damage evolution law (Lyakhovsky et al., 1997)

$$\frac{d\omega}{dt} = C_d I_2(\xi - \xi_0).$$

 I_1 and I_2 are invariants of the strain tensor, and the deformation state is

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Damage affects the elastic moduli

$$\lambda = \lambda_0 \ (+\dots), \qquad \mu = \mu_0 + \omega \xi_0 \gamma_r$$

(A) Basic decoupling Elasic relaxation

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(B) Damage increase

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(C) Failure at margin

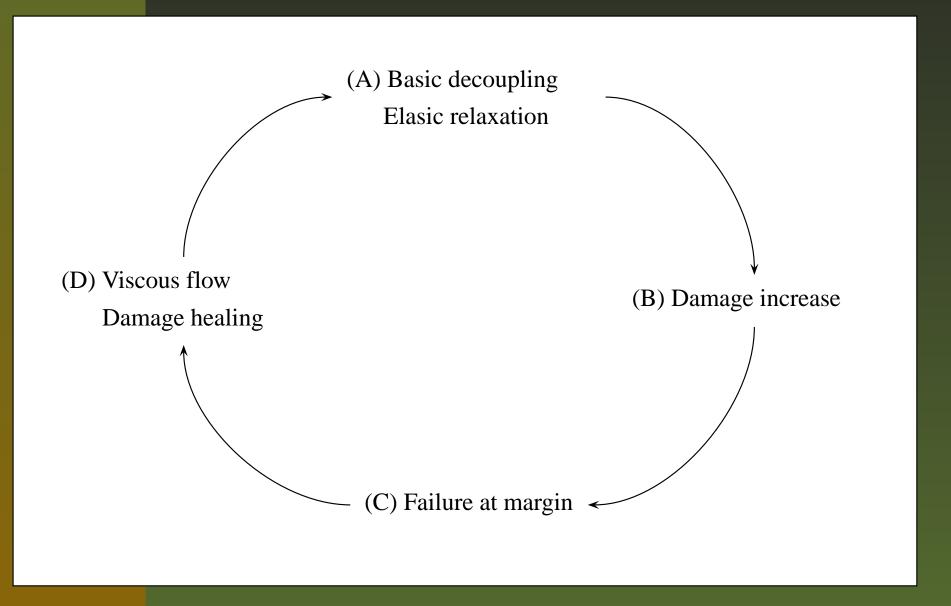
(A) Basic decoupling Elasic relaxation

(D) Viscous flow

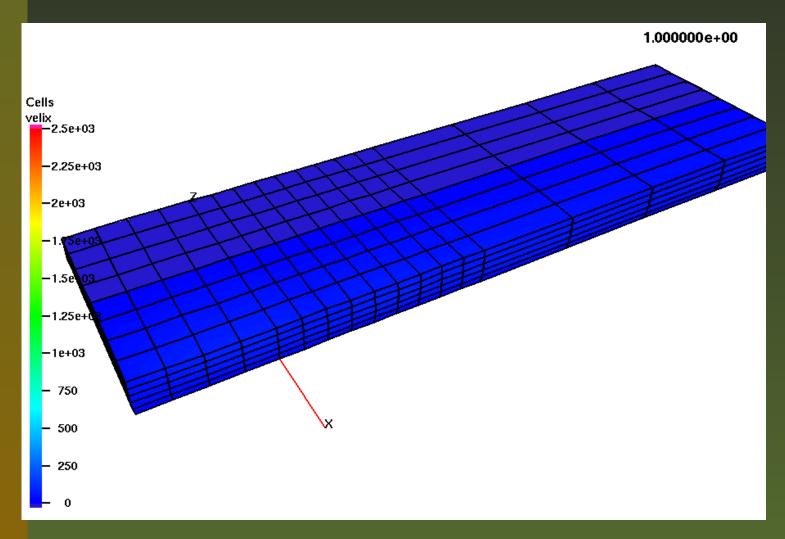
Damage healing

(B) Damage increase

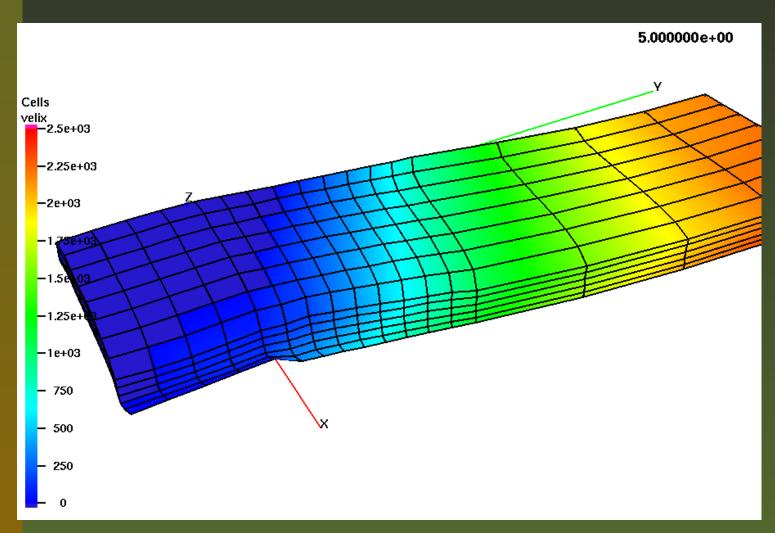
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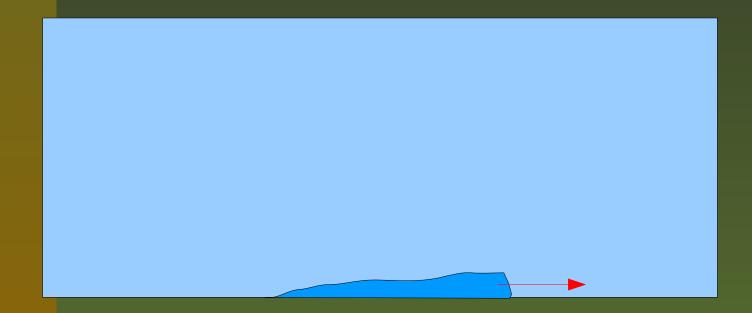


Motion events: the base

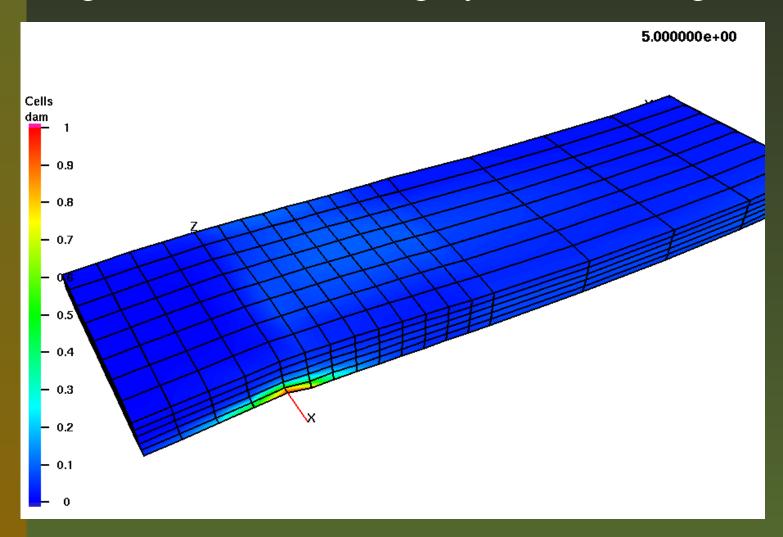
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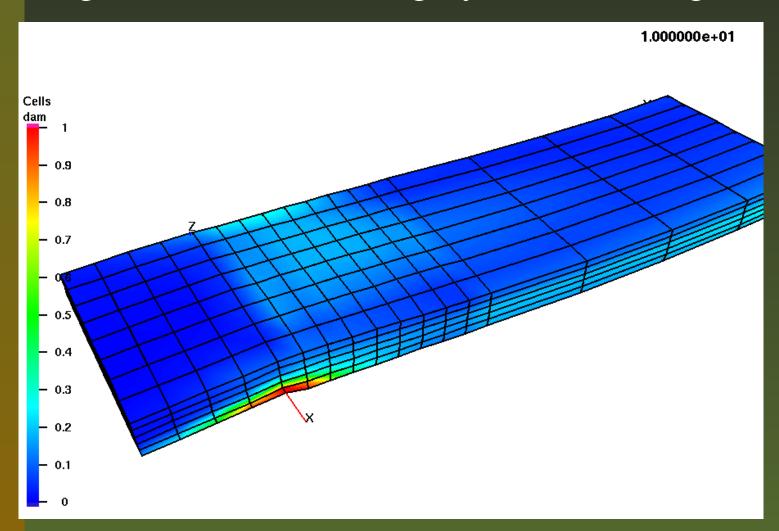
- (A₁) Failure of plastic till at the base, released by smaller tidal back-pressure.
- (A₂) Water-filled Yoffe-crack traveling along the ice-till interface (moving at subsonic velocity).



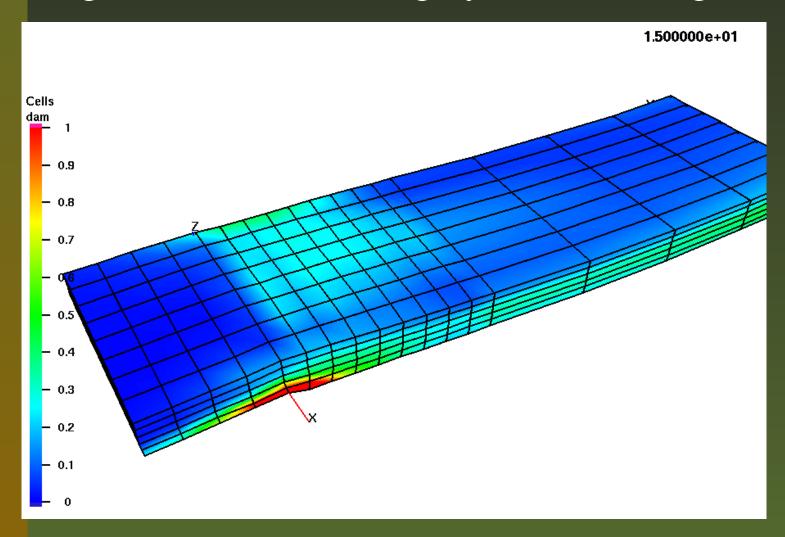
(B) Damage evolution at the highly stressed margin:



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- (C₂) Failure of the plastic till due to diffusing water.

 Rapid loading of the margins and cracking of the ice.

(D) Viscous flow, healing and damage reduction

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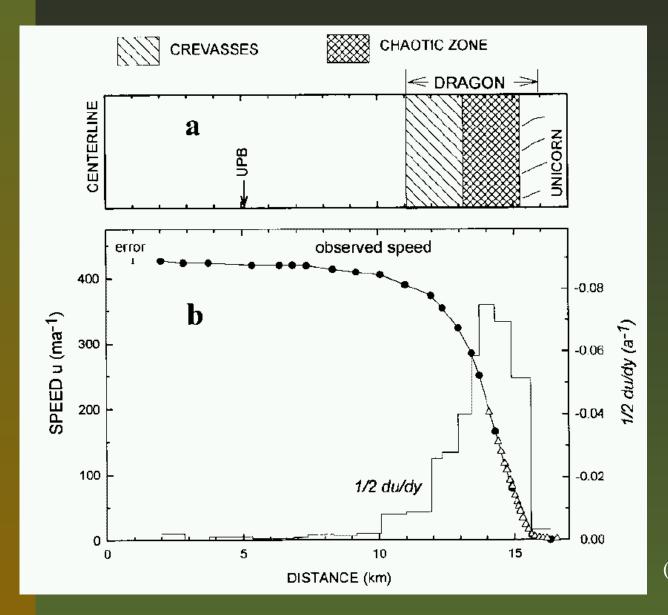
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Ice stream margins are reminiscent of active faults.

Damage mechanics has been successfully applied for

- prediction of fault location,
- prediction of margin migration,
- prediction of fault dynamics.

Velocity profile at margin (WIS)



(Echelmeyer et al., 1994)

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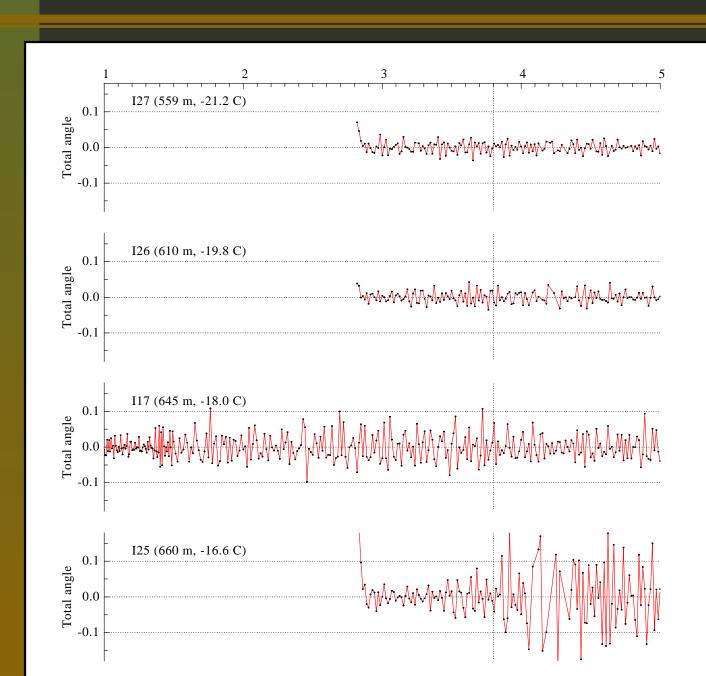
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Main problems:

- Formulation of damage law
- Constrain the parameters

Processes at depth (Jakobshavn margin)



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